Blind Compensation of Memoryless Nonlinear Effects in OFDM Transmissions Using CDF

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ABSTRACT

The high peak-to-average power ratio (PAPR) of orthogonal frequency-division multiplexing (OFDM) systems combined with nonlinear channel characteristics results in poor bit error rate (BER) performance and causes out-of-band spectrum regrowth in the channel. In order to combat the former effect, this paper proposes an efficient technique to compensate for memoryless nonlinear channel distortions by inserting at the receiver a discrete system designed with the aim of providing a linear equivalent channel. To derive the inverse function of the nonlinear channel, the proposed scheme is based on the transmitted and the received signal statistical properties without any explicit access to the transmitted signal. In particular, assuming the Rayleigh distribution of the OFDM signal envelope at the transmitter, the cumulative distribution function (CDF) at the channel output is estimated and used in the compensation at the receiver. To improve the compensation efficiency, the Gaussian kernel based CDF estimation is proposed. Computer simulations demonstrate that the proposed technique produces good system performance.

Categories and Subject Descriptors

B.4 [**Input/Output and Data Communications**]: Data Communications Devices;
C.2.1 [**Network Architecture and Design**]: Wireless communication

General Terms

Algorithms, Theory

Keywords

OFDM, nonlinear distortions, kernel based CDF estimation, peak-to-average power ratio

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1. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) has attracted considerable interest among communication system designers because of its robustness to multipath fading and simplified channel equalization (or no need for it). The application of the Inverse Fast Fourier Transform (IFFT) makes the implementation of this modulation scheme practical even at high data rates. In fact, OFDM has been adopted in several wireless standards such as IEEE 802.11a WLAN, digital terrestrial video broadcasting, digital audio broadcasting and HIPERLAN [1], [2], [3]. However, due to the large dynamic range of the modulated signal, OFDM is very sensitive to nonlinear distortions both in the high power amplifier (HPA) stages of the transmitter and in the channel. The nonlinearity causes (i) spectral-spreading of the OFDM signal and (ii) intermodulation between subcarriers which seriously degrade the system performance. To overcome the linearization challenges at the transmitter, several digital-predistortion schemes have been proposed [4], [5]. The basic idea behind these techniques relies on modeling the nonlinearity in HPA and its inverse function first and then passing the transmitted signal (before HPA) through the inverse nonlinearity (pre-distorter). As a result, the harmful effects introduced by the amplifier amplitude-to-amplitude (AM/AM) distortion could be reduced.

In this paper, the effects of the nonlinear channel are dealt with in the receiver which makes the task of compensating for nonlinearity more difficult. This is because the input signal to the nonlinearity is not available, and the construction of the simple look-up tables is not practical. This problem could be overcome by introducing training sequences as it is a case in the conventional equalization algorithms which OFDM does not require. Based on this premise, this paper introduces a novel approach to compensate the nonlinear distortions using a statistical approach. In particular, a new compensation method is proposed which exploits the Gaussian property of the complex OFDM signal with the large number of subcarriers. When the complex Gaussian OFDM signal with the Rayleigh envelope passes through a nonlinear channel, its inverse function is derived using the estimated CDF of the signal at the input to the receiver alone. This inverse function is then applied to the received signal to provide an overall linearization of the equivalent channel in the information bearing signal path. As a side effect, the compensating nonlinearity at the receiver makes the additive white Gaussian noise (AWGN) from the channel non-Gaussian and this may require some improved processing over matched filtering.

To estimate the CDF of the received signal, the empirical CDF, equivalent to histogram estimation of probability distribution function (PDF), could be employed [6]. However, the estimation accuracy of such an approach would heavily depend on the number of data samples available. In order to improve the compensation algorithm convergence, we propose a new Gaussian kernel based CDF estimation method. Computer simulation results confirm that the performance of the OFDM systems suffering from nonlinear channel distortions can be greatly improved using the proposed compensator with a minimum number of samples.

2. SYSTEM MODELING

In this section, we introduce the baseband OFDM function blocks and discuss the effects of nonlinearities in multicarrier systems.

2.1 OFDM System Description

Fig. 1 illustrates the baseband-equivalent functional block diagram of the OFDM transmission system with every block of the transmitter in the upper branch having a corresponding block in the receiver (lower branch).

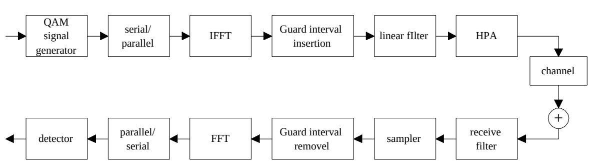

Figure 1: The simplified block diagram of the baseband-equivalent OFDM system.

The M-ary QAM signal generator produces $c_i = a_i + jb_i$ complex symbols with independent, identically distributed (i.i.d.) random in-phase and quadrature components a_i and b_i from the finite alphabet set $A = [\pm 1, \pm 3, \cdots, \pm\sqrt{M} - 1]$. The serial-to-parallel block converts the QAM input data stream into a block of N symbols, which in turn modulate the corresponding subcarrier. Through the $IFFT$ block, the baseband (discrete) OFDM signal is generated as:

$$x[k] = \sum_{n=0}^{N-1} c_n e^{j\frac{2\pi}{N}nk} k = 0, 1, \cdots, N-1, \tag{1}$$

where N is the number of subcarriers. When N is large, and because c_n's are I.I.D., from the Central Limit Theorem (CLT), the resulting OFDM complex signal $x[k]$ in (1) is a complex Gaussian process.

The transmit and the receive shaping filters, G_t and G_r respectively, have the frequency response:

$$|G_t(f)| = |G_r(f)| = \sqrt{G(f)}, \tag{2}$$

where $G(f)$ denotes a raised-cosine Nyquist pulse with a roll-off factor α.

The spectrally shaped signal at the output of the transmit filter is fed through the HPA into the channel. In this paper, the cascade of the HPA and the channel is modeled as a frequency independent memoryless nonlinear system, with $g(r)$ characterizing envelope AM/AM distortions as a function of the input magnitude r. We disregard the phase distortions. The linearization algorithms, developed in the next section, are generic enough to compensate any invertible $g(r)$ function, though in the simulation section the focus is on the practical traveling wave tube (TWT) amplifier nonlinearity [7], [8]:

$$g(r) = \frac{ar}{1 + \beta_\alpha r^2}, \tag{3}$$

where a is the small signal gain, $A_I = 1/\sqrt{\beta_\alpha}$ is the amplifier saturation voltage and $A_o = a/(2\sqrt{\beta_\alpha})$ stands for the maximum output amplitude. In order to describe the different output power levels, we use the output power backoff (OBO) of the HPA defined as:

$$OBO = 10 \log_{10} \frac{P_{max}}{P_o}, \tag{4}$$

where the P_{max} represents the maximum output power of the HPA at the saturation point and P_o denotes the mean output power of the signal at the HPA output.

2.2 Nonlinear Effects

The nonlinear effects on the OFDM signal in the frequency domain are illustrated in Fig. 2 where there is almost 25 dB power spectral density (PSD) regrowth in the PSD tail for the nonlinear system with the output backoff of 5.8 dB when compared to the linear system.

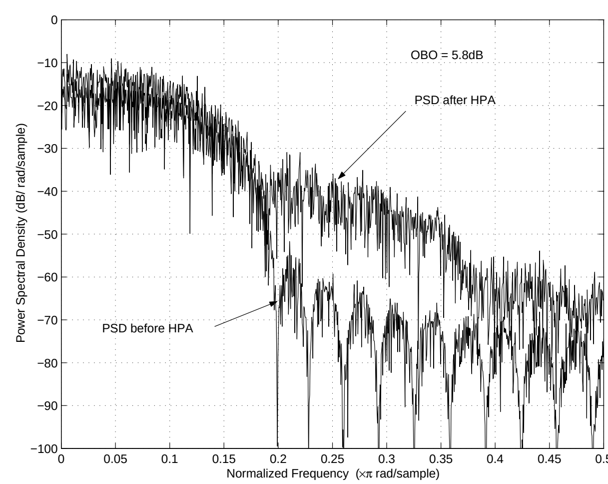

Figure 2: PSD of OFDM signal in systems with and without nonlinear distortions.

The effects of nonlinearity on the received 16-ary QAM constellations in the absence of the channel AWGN are shown in Figs. 3 and 4 which correspond to the linear and nonlinear systems, respectively. In the ideal case, there are 16 well defined points, even though we work with 10 times upsampling per symbol to simulate the continuous signals, and the raised cosine roll-off factor is $\alpha = 90\%$. In the nonlinear system, the received clouds have a circular shape at the signaling point which is characteristic to the AM/AM nonlinearities and is interpreted as the in-band noise. We refer to this problem as constellation warping.

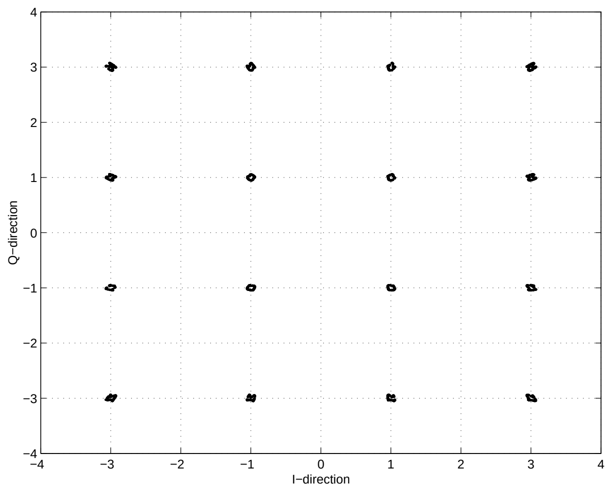

Figure 3: Received 16-ary QAM constellation with the linear channel.

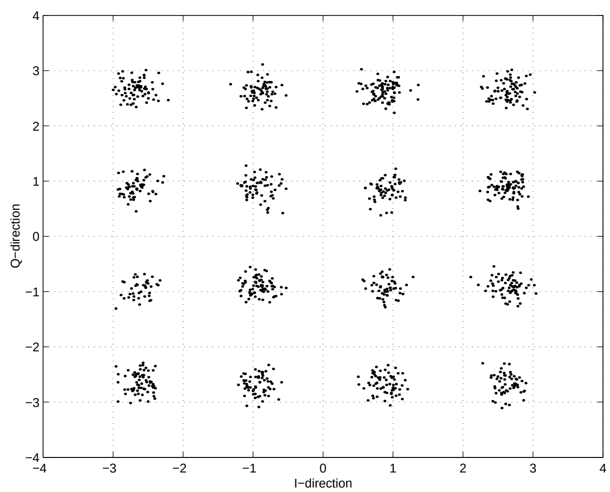

Figure 4: Received 16-ary QAM constellation with the nonlinear channel (HPA at OBO = 7.2 dB).

3. GAUSSIAN KERNEL BASED CDF ESTIMATION

Fig. 5 shows the block diagram of a proposed compensation technique. The proposed scheme is composed of: (i) the CDF estimator that computes the received signal CDF; and (ii) a compensator (inverse nonlinearity) that reduces constellation warping.

From the discussion in Section 2, the transmitted baseband-equivalent OFDM signal $x(k)$ is complex Gaussian and its envelope $r = |x(k)|$ follows Rayleigh distribution with the CDF as follows:

$$F_R(r) = 1 - e^{\frac{-r^2}{2\sigma^2}}, \tag{5}$$

where r is the OFDM signal amplitude and σ^2 is the variance of the complex Gaussian signals. When the Rayleigh envelope of the complex Gaussian OFDM signal passes through a nonlinear system characterized by the invertible function $g(\cdot)$,

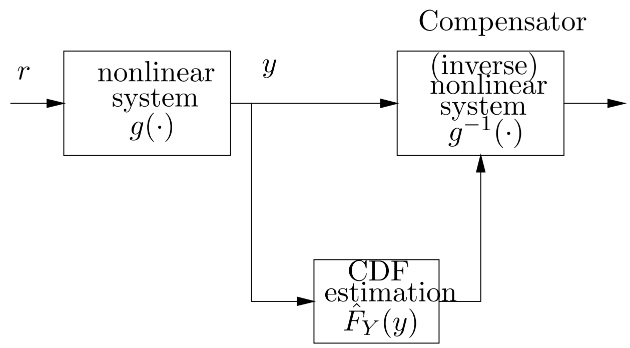

Figure 5: Block diagram of nonlinear distortion compensation.

according to the CDF definition, the CDF of the nonlinear system output envelope y is:

$$F_Y(y) = F_R(g^{-1}(y)), \tag{6}$$

where $F_R(\cdot)$ is the CDF distribution of the input envelope and $g^{-1}(\cdot)$ is the nonlinear system inverse function. Assuming that $F_Y(y)$ is monotonically increasing, from (6) the inverse function of the nonlinear system can be written as:

$$g^{-1}(y) = F_R^{-1}(F_Y(y)). \tag{7}$$

Based on (5), the inverse function $F_R^{-1}(r)$ is:

$$F_R^{-1}(r) = \sqrt{-2\sigma^2 \ln(1-r)}. \tag{8}$$

Substituting (8) into (7), the nonlinear system inverse transfer function $g^{-1}(\cdot)$ is given as:

$$g^{-1}(y) = \sqrt{-2\sigma^2 \ln(1-F_Y(y))}, \tag{9}$$

where the y and $F_Y(y)$ are the nonlinear system output signal envelope and its CDF.

To obtain the inverse function of the nonlinear system based on (9) requires an efficient method to estimate $F_Y(y)$. The first order approximation of the CDF is the empirical CDF but its estimation accuracy heavily depends on the number of samples. In order to improve the utility of samples, we propose a new Gaussian kernel based CDF estimation method. The kernel estimator for the PDF, $\widehat{f}(x)$, with the kernel function $k(\cdot)$ is defined as [9]:

$$\widehat{f}(x) = \frac{1}{n \cdot h} \sum_{i=1}^{n} k\left(\frac{x - x_i}{h}\right), \tag{10}$$

where h is the window width, also called the smoothing parameter, and $x_i, i = 1, 2, \cdots n$ are the data samples, whose underlying PDF, $f(\cdot)$, is to be estimated. In the kernel based estimation, the kernel function $k(\cdot)$ satisfies the condition:

$$\int_{-\infty}^{+\infty} k(x)dx = 1 \tag{11}$$

There are lots of kernel functions which match equation (11). One of the most accepted kernels, used in this paper, is the Gaussian kernel defined as:

$$k(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}. \tag{12}$$

Substituting (12) into (10), we get the Gaussian kernel PDF estimator:

$$\widehat{f}(x) = \frac{1}{\sqrt{2\pi} nh} \sum_{i=1}^{n} e^{-\frac{(x-x_i)^2}{2h^2}}. \tag{13}$$

Integrating (13), the Gaussian kernel CDF estimator $\widehat{F}(x)$ is:

$$\widehat{F}(x) = \frac{1}{2} + \frac{1}{2n} \sum_{i=1}^{n} \text{erf}\Big(\frac{x - x_i}{\sqrt{2}h}\Big), \tag{14}$$

where $\text{erf}(\cdot)$ denotes the error function:

$$\text{erf}(x) = \frac{1}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{15}$$

Combining (14) and (9), the proposed procedure for calculating the inverse function using the Gaussian kernel CDF is based on the following formula:

$$g^{-1}(y) = \sqrt{-2\sigma^2 \ln(1 - (\frac{1}{2} + \frac{1}{2n} \sum_{i=1}^{n} \text{erf}(\frac{y - y_i}{\sqrt{2}h})))} \tag{16}$$

where $y_i, i = 1, 2 \cdots, n$ are the received signal envelope samples.

The inverse function computation in (9) and (16) as well as its accuracy depend strongly on the estimation accuracy in the proposed Gaussian kernel CDF method. In turn, the Gaussian kernel CDF estimation accuracy depends on the number of data samples and the kernel window width. Fig. 6 shows the estimated PDF for two different kernel window widths with the same number of Gaussian distributed data points ($n = 15$). It is evident from this figure that the choice for the window width h is critical to improve the estimation accuracy. According to [9], to minimize the PDF approximation mean square error, for heavily skewed and long tail distribution data, the optimal window width h_{opt} should be chosen as:

$$h_{opt} = 0.79 R n^{-1/5}, \tag{17}$$

where R is the interquartile range of the underlying distribution. Similarly, we found that this choice for h results in acceptable performance of the proposed Gaussian kernel CDF estimators.

4. SIMULATION RESULTS

To demonstrate the performance of the proposed linearization scheme, we evaluated the BER using Monte Carlo simulations for systems with the empirical and the Gaussian kernel CDF estimators. For comparison purposes, we also show

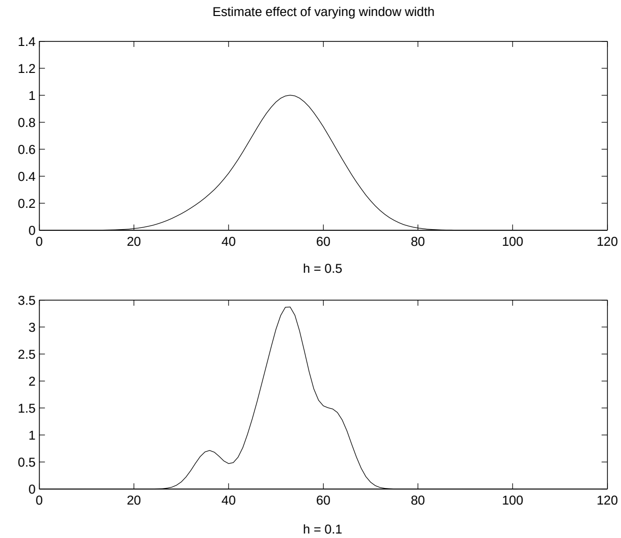

Figure 6: Effects of window widths on the Gaussian kernel PDF estimation accuracy.

the performance for systems without linearizers and systems with ideal (linear) channels. The BER of the OFDM system without nonlinear distortions in the AWGN channel is the same as that of the corresponding narrowband M-ary QAM system and is used here for benchmarking the performance of different schemes. The nonlinear channel model adopted in simulations is based on (3) with different values for OBO: the lower the OBO, the more severe nonlinearity of the channel. The simulations were carried out for the OFDM system with 1024 subcarriers and 16-ary QAM signaling on each subcarrier with 10 samples per symbol.

Figs. 7 and 8 show the BER as the function of SNR ($\frac{E_b}{N_0}$) for linearizers with the Gaussian kernel and the empirical CDF estimators, respectively. In both figures, OBO =5.7 dB, and the number of points used to estimate CDF is 100. Fig. 9 shows the BER performance of the empirical CDF based linearizer, when the OBO is 4.6 dB, and the number of points used to estimate CDF is 10,000. All figures demonstrate that the BER performance is severely degraded due to nonlinear distortions in systems without nonlinearity compensation. Based on Figs. 7 and 8, when the number of points used to estimate the CDF is in the range of 100, for 16-ary QAM OFDM, the Gaussian kernel CDF linearizer achieves 0.5 dB and 1 dB gain improvements over the empirical CDF linearizer for $BER = 10^{-2}$ and $BER = 10^{-4}$, respectively. When the number of training points is lower and/or the M-ary QAM constellation level is higher, these improvements are even more evident. When the number of points used to estimate CDF is large, (as in Fig. 9), the performance of the empirical CDF compensator is comparable to that of the more complex Gaussian kernel CDF compensator.

5. CONCLUSIONS

In this paper, we have proposed and analyzed a receiver-based linearizer which compensates for an arbitrary, invertible, channel nonlinearity in radio systems employing OFDM. The linearizer operates in the blind mode as it only assumes Rayleigh distribution for the envelope of the transmitted signal and utilizes estimated CDF of the received data to construct the channel inverse nonlinearity. The algorithm exhibits robust performance and rapid convergence by utilizing a novel Gaussian kernel CDF estimation procedure. In principle, using the proposed method it is possible to compensate for a nonlinear channel to any desired degree of accuracy provided a float point arithmetic is employed. In the information signal path, because of complexity, the presented approach is more useful when the uncompensated channel is strongly nonlinear and simple CDF estimation methods do not work. However, for the channel AWGN path, which after passing through the compensating nonlinearity is becoming non-Gaussian, the issue of detecting OFDM symbols in the optimum way remains an open problem. The proposed scheme may be combined with the adaptive power back-off strategies and PSD code shaping at the transmitter to achieve even higher linearization gains.

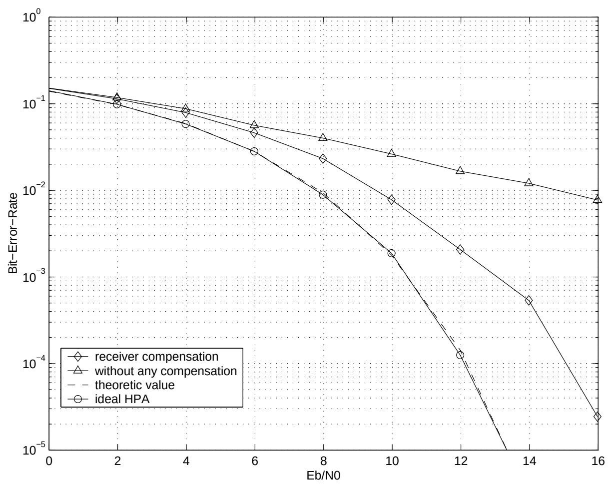

Figure 7: BER versus E_b/N_0 for the Gaussian kernel CDF linearizer with OBO =5.7 dB and 100 training points.

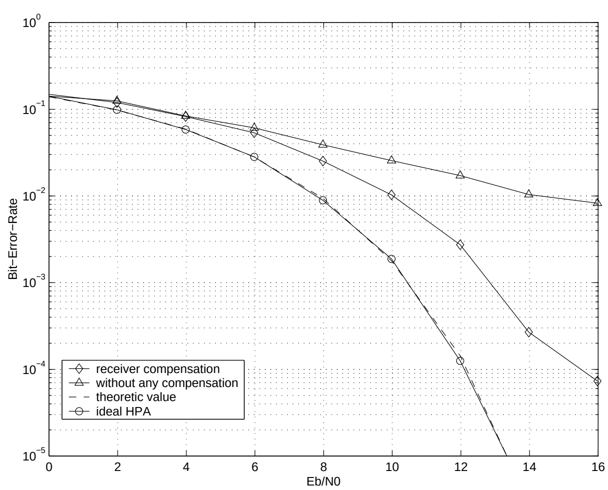

Figure 8: BER versus E_b/N_0 for the empirical CDF linearizer with OBO =5.7 dB and 100 training points.

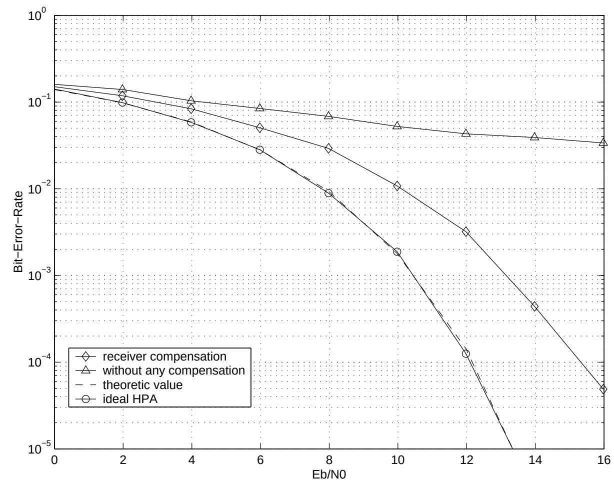

Figure 9: BER versus E_b/N_0 for the empirical CDF linearizer with OBO =4.6 dB and 10,000 training points.

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