Frequency Domain Equalization with Multiple Receiving Antennas for Single Carrier Signaling

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ABSTRACT

This paper presents a space-time (single-input and multiple output (SIMO)) transceiver scheme for reliable transmissions over frequency selective fading channels using conventional single-carrier M-ary PSK or QAM modulations. The main feature of this scheme is in appending to the transmitted signaling frame a circular suffix segment so that by exploiting the spectral structure of this kind of signaling the inter-symbol interference (ISI) effects could be fully mitigated at the receiver using efficient processing. It is demonstrated through Monte-Carlo simulations that the performance of this scheme over time-dispersive channels is comparable to that of SIMO-OFDM. OFDM has high peak to average power ratio (PAPR) and demands the use of highly linear power amplifiers (PA's). The proposed scheme offers the advantage of reduced constraints on the PA linearity and signaling synchronization, which is critical in wireless applications.

Categories and Subject Descriptors

B.4 [**Input/Output and Data Communications**]: Data Communications Devices; C.2.1 [**Network Architecture and Design**]: Wireless communication

General Terms

Algorithms, Theory

Keywords

Multiple antenna systems, blind channel equalization

1. INTRODUCTION

The ever-increasing transmission rate of wireless applications places demands on the equalization algorithms to be even more efficient in terms of convergence, complexity and adaptability depending on the modulation scheme employed. For high-rate wideband communications, the channel time-spreading could be very large compared to a symbol time slot. Conventional time-domain decision-feed-back equalizers (DFE) have to have tens or more than a hundred of taps to equalize such kind of channels.

In the last ten years, owing to its unique signaling structure to facilitate efficient channel equalization, OFDM became one of the most prominent schemes in time-dispersive channels [1] [2]. However, the OFDM advantages in channels impaired by ISI are offset by its high peak-to-average power ratio (PAPR). It has been demonstrated that OFDM performance depends highly on the linearity of the whole transmission link [4]. Nonlinear distortions of the channel or transceiver cause dramatic OFDM performance degradation as they give rise to both severe ISI and inter-carrier-interference (ICI).

The single carrier frequency domain equalization (SCFDE) scheme [5] employs the conventional M-ary PSK or QAM single-carrier signaling and has comparable performance over time-dispersive fading channels as OFDM. This paper presents a SIMO-SCFDE transceiver scheme in which multiple receive antennas are deployed. The proposed scheme has a favorable tradeoff between the channel equalization performance and the processing complexity. This scheme has the advantage that it does not face the high PAPR problem as OFDM, and has a same overall transceiver complexity as that of SIMO-OFDM. This scheme is practical and capable of fully removing ISI via efficient processing in the frequency domain. It is frame-based and could be employed in a semi-blind fashion. Its signaling frames possess a circular suffix segment and the receiver exploits the spectral structure of circular-shifted sequences. It is this special signaling frame structure that facilitates the ISI mitigation at the receiver.

2. TRANSMISSION SIGNALING

In SCDFE, N information-bearing M-ary PSK or QAM symbols $s_j^{[k]},\ j = 1, \cdots, N$ are assumed to be carried in the k-th frame time slot $\mathbf{S}^{[k]} = [s_1^{[k]}, s_2^{[k]}, s_3^{[k]}, \ldots, s_N^{[k]}]$. The transmission procedure is illustrated in Fig.1. Rather than transmitting the data vector $\mathbf{S}^{[k]}$, G circular-shift-suffix symbols (or fixed symbol sequence) are appended following the data symbols in the guard time interval T_G. Fixed symbol sequence appending is favorable for the purposes of semiblind channel estimation, tracking, and synchronization [6]. Without losing generality, we assume circular-shift-suffix is transmitted here. Hence, what enters the channel is the

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frame defined as:

$\mathbf{F}^{[k]} = [\overbrace{s_1^{[k]}, s_2^{[k]}, s_3^{[k]}, s_4^{[k]}, s_5^{[k]}, \cdots s_N^{[k]}, s_1^{[k]}, s_2^{[k]}, \cdots, s_G^{[k]}}^{W}]$ where $W = N + G$ is the frame length; $G = \frac{T_G}{T_s} \geq L$; and $L+1$ is the maximum length of the multiple channels. The transmitted signal in the k^{th} frame from a transmit antenna is modulated with a single carrier:

$s(t) = Re\{\sum_{w=1}^{W} g(t-(w-1)T_s - kT_f)$
$\times \mathbf{F}_{(w)}^{[k]} e^{j2\pi f_c(t-(w-1)T_s-kT_f)}\}$

where $g(t)$ is a shaping pulse, f_c, T_f, T_s are the carrier's frequency, the time interval for a frame and the time slot for a symbol, respectively.

3. RECEIVER ALGORITHM FOR SIMO CASES

R antennas are assumed to be employed at the receiver. For a quasi-static discrete-time modeling of single input and multiple output (SIMO) channels, a matrix $\mathbf{H}$ is used to capture the impulse responses of the multiple channels [9]:

$$\mathbf{H}_{R\times(L+1)} = \begin{bmatrix} h_1^{[1]} & h_1^{[2]} & \cdots & h_1^{[L+1]} \\ h_2^{[1]} & h_2^{[2]} & \cdots & h_2^{[L+1]} \\ \vdots & \vdots & \vdots & \vdots \\ h_R^{[1]} & h_R^{[2]} & \cdots & h_R^{[L+1]} \end{bmatrix} = \begin{bmatrix} \mathbf{h}(1) & \mathbf{h}(2) & \cdots & \mathbf{h}(L+1) \end{bmatrix} \quad (1)$$

where $h_q^{[i]}$ represents the gain from the transmit antenna to the q^{th} receive antenna, and i is a delay index. In the proposed scheme, the received signals, after down-converting and base-band filtering, are sampled at the symbol rate and arranged in a matrix $\mathcal{Y}^{[k]}$ (within the k^{th} frame interval between t_1 to t_2 as illustrated in Fig.1), samples from the same receive antenna are in one row. Without noise, the sampled data matrix $\mathcal{Y}^{[k]}$ is a convolution of discrete-time channel impulse response with the transmitted signals. In a matrix format, this relationship could be expressed as follows:

$$\mathcal{Y}_{R\times W}^{[k]} = \mathbf{H}_{R\times(L+1)}\mathcal{X}_{(L+1)\times W}^{[k]} + \mathcal{N}_{R\times W} \quad (2)$$

where $\mathcal{N}$ stands for the noise component matrix,

$$\mathcal{Y}_{R\times W}^{[k]} = \begin{bmatrix} y_{(1,1)}^{[k]} & y_{(1,2)}^{[k]} & \cdots & \cdots & y_{(1,W)}^{[k]} \\ y_{(2,1)}^{[k]} & y_{(2,2)}^{[k]} & \cdots & \cdots & y_{(2,W)}^{[k]} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{(R,1)}^{[k]} & y_{(R,2)}^{[k]} & \cdots & \cdots & y_{(R,W)}^{[k]} \end{bmatrix}$$

$$\mathcal{X}_{(L+1)\times W}^{[k]} = \begin{bmatrix} \mathbf{F}_{(1)}^{[k]} & \mathbf{F}_{(2)}^{[k]} & \cdots & \cdots & \cdots & \mathbf{F}_{(W)}^{[k]} \\ \mathbf{F}_{(W)}^{[k-1]} & \mathbf{F}_{(1)}^{[k]} & \cdots & \cdots & \cdots & \mathbf{F}_{(W-1)}^{[k]} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{F}_{(W-L+1)}^{[k-1]} & \cdots & \mathbf{F}_{(W)}^{[k-1]} & \mathbf{F}_{(1)}^{[k]} & \cdots & \mathbf{F}_{(W-L)}^{[k]} \end{bmatrix}$$

The first processing of proposed receiver on the received data matrix $\mathcal{Y}^{[k]}$ is to remove the first G columns so that a shorter data matrix $\mathbb{Y}^{[k]}$ is obtained:

$$\mathbb{Y}_{R\times N}^{[k]} = \mathbf{H}_{R\times(L+1)}\mathbb{X}_{(L+1)\times N}^{[k]} + \mathbb{N}_{R\times N} \quad (3)$$

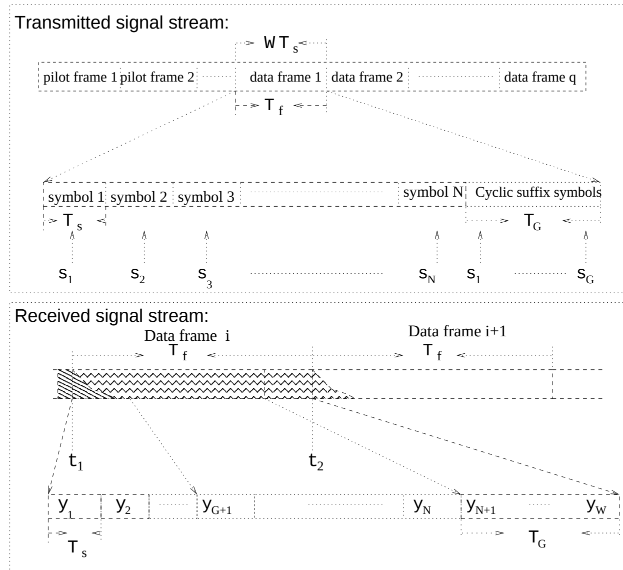

Figure 1: The timing of the transmitted and received signal streams.

$$\mathbb{Y}_{R\times N}^{[k]} = \begin{bmatrix} y_{(1,G+1)}^{[k]} & y_{(1,G+2)}^{[k]} & \cdot & \cdot & y_{(1,W)}^{[k]} \\ y_{(2,G+1)}^{[k]} & y_{(2,G+2)}^{[k]} & \cdot & \cdot & y_{(2,W)}^{[k]} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{(R,G+1)}^{[k]} & y_{(R,G+2)}^{[k]} & \cdot & \cdot & y_{(R,W)}^{[k]} \end{bmatrix}$$

$$\mathbb{X}_{(L+1)\times N}^{[k]} = \begin{bmatrix} \mathbf{F}_{(G+1)}^{[k]} & \mathbf{F}_{(G+2)}^{[k]} & \cdots & \cdots & \mathbf{F}_{(W)}^{[k]} \\ \mathbf{F}_{(G)}^{[k]} & \mathbf{F}_{(G+1)}^{[k]} & \cdots & \cdots & \mathbf{F}_{(W-1)}^{[k]} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{F}_{(G-L+1)}^{[k]} & \mathbf{F}_{(G-L+2)}^{[k]} & \cdots & \cdots & \mathbf{F}_{(W-L)}^{[k]} \end{bmatrix} \quad (4)$$

Furthermore, from (1) and (3), $\mathbb{Y}_{R\times N}^{[k]}$ is expressed as follows:

$$\mathbb{Y}_{R\times N}^{[k]} = \sum_{i=1}^{L+1} \mathbf{h}_{R\times 1}(i)\mathbb{X}_{(i,:)}^{[k]} + \mathbb{N}_{R\times N} \quad (5)$$

where $\mathbb{X}_{(i,:)}$, the i^{th} row of $\mathbb{X}$, is a circularly shifted version of $\mathbf{S}^{[k]}$. This fact can be observed from matrix (4) and the definition of $\mathbf{F}^{[k]}$. Hence,

$$\mathbb{Y}_{R\times N}^{[k]} = \sum_{i=1}^{L+1} \mathbf{h}(i)\mathcal{S}_{(G-i+1)}^{[k]} + \mathbb{N} \quad (6)$$

where $\mathcal{S}_{(i)}^{[k]}$ stands for $\mathbf{S}^{[k]}((n+i))_N$, a left circularly shifted version of $\mathbf{S}^{[k]}(n)$. The following aspects and theorem about Discrete Fourier Transform (DFT) and Inverse Discrete Fourier Transform (IDFT) are useful for further analysis of the signal structure and the derivation of the receiver algorithm. If $\mathbf{x} = [x(1), x(2), x(3), \cdots, x(N)]$ and $\mathbf{X} = [X(1), X(2), X(3), \cdots, X(N)]$ has the relationship that: $x(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} \mathbf{X}(n)$, the linear transformation formats of DFT and IDFT are:

$$\textbf{DFT:}\quad \mathbf{X}_N = \mathbf{x}_N\mathbf{V}, \qquad \textbf{IDFT:}\quad \mathbf{x} = \frac{1}{N}\mathbf{X}_N\mathbf{V}^H \tag{7}$$

$$\mathbf{V}_{N\times N} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & v & v^2 & \cdots & v^{N-1} \\ 1 & v^2 & v^4 & \cdots & v^{2(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & v^{N-1} & v^{2(N-1)} & \cdots & v^{(N-1)(N-1)} \end{bmatrix}$$

$\mathbf{V}_{N\times N}$ is a Vandermonde matrix with $v = e^{-j2\pi/N}$ [8]. In this case, $\mathbf{V}$ is also a Fourier transform matrix.

Theorem 1. *[8]*

If $x(n)\underset{N}{\overset{DFT}{\longleftrightarrow}}X(k)$, *then,* $x((n-l))_N\underset{N}{\overset{DFT}{\longleftrightarrow}}X(k)e^{-j2\pi kl/N}$

From Theorem 1, it could be concluded that:

$$\mathcal{S}^{[k]}_{(i)} = \mathcal{IDFT}((\mathbf{Q}(i)\odot\mathcal{DFT}(\mathbf{S}^{[K]}))) \tag{8}$$

where $\odot$ stands for element by element product and

$$\mathbf{Q}(i) = [e^{j2\pi i/N}, e^{j2\pi 2i/N}, e^{j2\pi 3i/N}, \cdots, e^{j2\pi Ni/N}] \tag{9}$$

hence,

$$\mathcal{S}^{[k]}_{(G-i+1)} = \mathbf{Q}(G-i+1)\mathbb{S}^{[k]} \tag{10}$$

where

$$\mathbb{S}^{[k]} = \frac{1}{N}\begin{bmatrix} \gamma_1^{[k]}, & 0 & \cdots & 0 \\ 0 & \gamma_2^{[k]} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \gamma_N^{[k]} \end{bmatrix}\mathbf{V}^H \tag{11}$$

and $[\gamma_1^{[k]}, \gamma_2^{[k]}, \gamma_3^{[k]}, \ldots \gamma_N^{[k]}] = [S_1^{[k]}, S_2^{[k]}, S_3^{[k]}, \ldots, S_N^{[k]}]\mathbf{V}$.

From (6) and (10) ,

$$\mathbb{Y}^{[k]}_{R\times N} = \sum_{i=1}^{L+1}\mathbf{h}(i)\mathbf{Q}(G-i+1)\mathbb{S}^{[k]} + \mathbb{N}$$

and this relation in matrix form is:

$$\mathbb{Y}^{[k]}_{R\times N} = \mathbf{C}\mathbb{S}^{[k]} + \mathbb{N} \tag{12}$$

where $\mathbf{C} = \sum_{i=1}^{L+1}\mathbf{h}(i)\mathbf{Q}(G-i+1) = \mathbf{H}\mathcal{Q}$, and

$$\mathcal{Q} = [\mathbf{Q}(G)^T, \mathbf{Q}(G-1)^T, \mathbf{Q}(G-2)^T, \cdots, \mathbf{Q}(G-L)^T]^T.$$

The signaling structure described by (8) and (12) facilitates direct and efficient processing to be conducted in frequency domain to achieve channel equalization and the symbol detection. The parameter matrix $\mathbf{C}$ can be expressed as:

$$\mathbf{C} = \begin{bmatrix} \mathcal{C}_{11} & \mathcal{C}_{12} & \cdots & \mathcal{C}_{1N} \\ \mathcal{C}_{21} & \mathcal{C}_{22} & \cdots & \mathcal{C}_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \mathcal{C}_{R1} & \mathcal{C}_{R2} & \cdots & \mathcal{C}_{RN} \end{bmatrix} \tag{13}$$
$$= [\mathbf{C}(:,1), \mathbf{C}(:,2), \cdots \mathbf{C}(:,N)]$$

where $\mathbf{C}(:,i)$ is the column vector of $\mathbf{C}$. From (12), it is concluded that:

$$\mathbb{Y}^{[k]}_{R\times N}\mathbf{V} = [\gamma_1^{[k]}\mathbf{C}(:,1), \gamma_2^{[k]}\mathbf{C}(:,2), \cdots, \gamma_N^{[k]}\mathbf{C}(:,N)] + \mathbb{N}\mathbf{V} \tag{14}$$

Therefore, the estimation of $\gamma_i^{[k]}$ via maximum ratio combining method can be expressed as follows:

$$\widehat{\gamma}_i^{[k]} = \mathbf{C}(:,i)^H[\mathbb{Y}^{[k]}_{R\times N}\mathbf{V}]_{(:,i)}/(\mathbf{C}(:,i)^H\mathbf{C}(:,i))$$

Eventually,

$$\begin{aligned}\widehat{\mathbf{S}}^{[k]} &= [\hat{s}_1^{[k]}, \hat{s}_2^{[k]}, \cdots, \cdots, \hat{s}_N^{[k]}] \\ &= \mathcal{IDFT}\{[\widehat{\gamma}_1^{[k]}, \widehat{\gamma}_2^{[k]}, \widehat{\gamma}_3^{[k]}, \ldots \widehat{\gamma}_N^{[k]}]\}.\end{aligned} \tag{15}$$

There are numbers of methods to estimate the channel via blind or semiblind methods including their adaptive versions for time-varying channels. Full discussion on how to apply these methods in the proposed scheme is beyond the scope of this paper. Owing to the nature of the SCFDE scheme, there is an efficient method to estimate the channel impulse response (CIR) of SIMO channels. The approach for estimating CIR with linear minimum mean-squared estimation is:

$$\mathbf{H} = \mathbf{C}\mathcal{Q}^H(\mathcal{Q}\mathcal{Q}^H)^{-1}, \tag{16}$$

where

$$\begin{aligned}\mathbf{C} &= \mathbf{E}(\mathbb{Y}^{[k]}(\mathbb{S}^{[k]})^{-1}) \\ &= \mathbf{E}\left(\mathbb{Y}^{[k]}\mathbf{V}\begin{bmatrix} \frac{1}{\gamma_1^{[k]}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\gamma_2^{[k]}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \\ 0 & 0 & \cdots & \frac{1}{\gamma_N^{[k]}} \end{bmatrix}\right)\end{aligned} \tag{17}$$

and $\mathbf{E}$ stands for the statistical mean. Thus, $\mathbf{H}$ can be estimated with the pilot signal aid. From (17), it is clear that the DFT of the pilot signal sequence should not have zero values. It is feasible to make use of detected signal frame in updating $\mathbf{H}$ so that the channel variance could be tracked.

4. PERFORMANCE SIMULATIONS

In order to verify the performance of SCFDE in multipath fading environments, a large number of Monte Carlo simulations have been conducted to obtain the Bit Error Rate (BER) as a function of Signal-to-Noise Ratio. The typical BER results and constellations before and after equalization are illustrated in Fig.2 Fig.3, and Fig.4 . Corresponding to the curves in these figures, the maximum number of symbols contributing to the ISI was assumed to be $L = 5$. Parameters $R = 1, 2, 3$ and $G = 5$ were respectively assumed for different simulations. The transmitted symbol constellations are respectively QPSK and 16-ary QAM. The simulation results were statistically averaged over all the cases of random multipath delays, random channel states, random bit streams and random additive Gaussian noise components.

For comparison purposes, the performances of QPSK-OFDM and 16QAM-OFDM were simulated with the same transceiver antenna setup, frame guard time interval, and time-dispersive fading channels. The results are reported in Fig.3 and Fig.4 with curves being labeled as OFDM. It can be observed the SCFDE achieved a robust performance for SISO and SIMO fading channels which are comparable to the performances of OFDM schemes with same transmission rates over same time-dispersive channels.

There is variety of other configurations for SCFDE in terms of receiver antennas, modulation levels, frame lengths and guard time intervals. These parameters should be properly chosen by taking into account channel time-dispersive length, SNR and time-varying characteristics of the channel.

5. CONCLUSIONS

This paper proposes a new efficient transceiver framework for time-dispersive fading channels. This scheme achieves comparable performance to that of OFDM. Because the scheme is based on the conventional modulations, it does not require highly linear PA as in the case of OFDM. With a proper choice of parameters W, N, T_s and the guard time T_G, SCFDE is able to fully mitigate the multipath time-dispersive impairments. The simulations demonstrate (i) the SCFDE's robust performance over multi-path fading channels and (ii) the scheme is not sensitive to the different channel lengths provided the $T_G > LT_s$.

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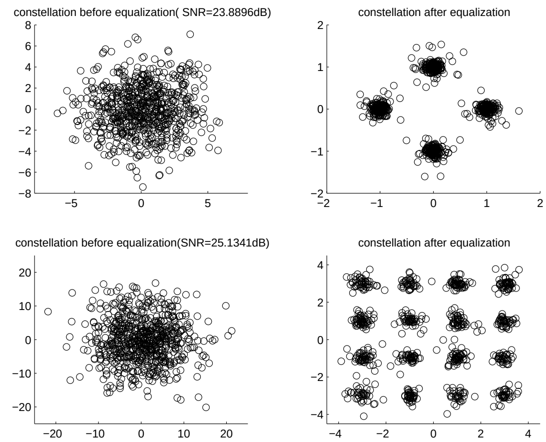

Figure 2: Symbol constellations before and after equalization (QPSK and 16-ary QAM SCFDE)

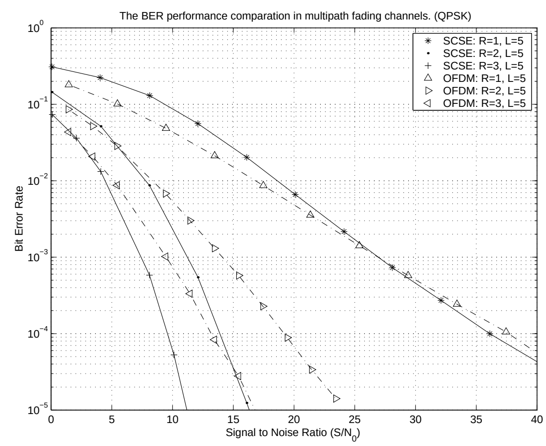

Figure 3: The BER performance over multipath fading channel (L=5, QPSK-SCFDE, QPSK-OFDM)

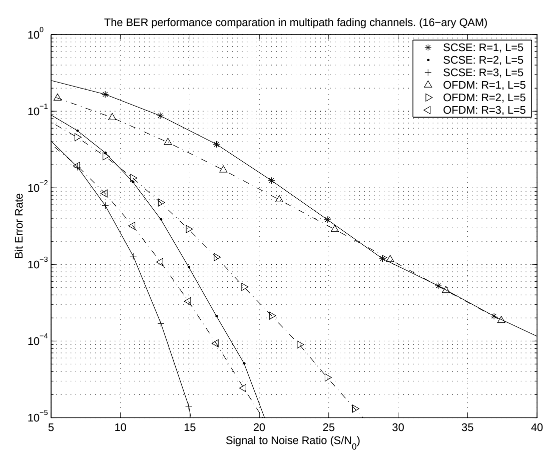

Figure 4: The BER performance over multipath fading channel (L=5, 16-ary, QAM-SCFDE, QAM-OFDM).